Swarm Splitting and Multiple Targets Seeking in Multi-Agent Dynamic Systems

Zhifu Chen, Tianguang Chu, and Jianlei Zhang

Abstract—This paper presents an approach to swarm split control of a system of multi-agents with limited sensing capabilities. The control scheme utilizes the competition between the inter-agent repulsive and attractive interactions and can split one cohesive swarm into several clustered subswarms along the direction perpendicular to the common heading direction of agents. The cohesion and collision avoidance of agents are ensured by long-range attractive and short-range repulsive interactions between agents. The split of swarm is achieved via a Gaussian-like repulsive interaction between agents, whose magnitude affects the number of subswarm clusters and can be designed to control the swarm splitting/rejoining maneuver, and whose maximum location mainly affects the relative distance between clustered subswarms. The control split law is also applied to double targets seeking task in a swarm of 100 agents, and simulations are worked out. These results are of interest in understanding and utilizing the splitting dynamics in swarms of agents with local coupling interactions.

I. INTRODUCTION

In recent years, the study of swarming behavior of large number of interacting individuals has attracted great attentions in many fields because of the prevalence of collective and cooperative phenomena in nature, such as fish schools, and bird flocks etc. [1]. It is believed that the underlying principles and mechanism of such behavior may give us useful ideas for developing multi-agent systems (MASs), such as mobile sensors, unmanned aerial vehicles (UAV), autonomous robot teams and so on.

One of the main issues in the study addresses the pattern formation control of MASs by means of limited information and interaction between agents [2]–[12]. Particularly, how to make a coherent swarm split into clustered subswarms (or subgroups), as illustrated in Figure 1, is a fascinating problem of both theoretical and practical interests. A fish school may split to evade predators, and a flock of birds can split into smaller foraging flocks and then rejoin again [13], [14]. For a robot team, it is a natural idea to split the team into groups for avoidance of obstacles or searching tasks. Also, in the problem of multiple targets tracking in mobile sensor network, it often requires some sensors to split from the existing formation to track new targets [15], [16]. In addition, other factors like the need of distinction or segregation of agents of different types, and the presence of adversarial agents, may lead to splitting in MASs as well.

In [17], a split control scheme is presented for MASs, which makes use of a homotopy parameter to switch between global and local couplings. The splitting realizes as a result of the relative strong coupling between leader agents and follower agents, and relative weak coupling between follower agents. If two or more leaders in the system diverge, then the followers will follow their closest leaders respectively, making the coherent swarm be broken into a few subswarms. For UAV swarm mission planning, parallel simulation algorithms are developed in [18] for subswarms with prescribed members going out in search of their individual targets. In [19], algorithms for flocking, fragmentation, split/rejoin maneuver are designed for MASs. Of which the fragmentation appears as the system failed to form flock under some initial conditions, and the splitting/rejoining is due to the presence of obstacles. For a swarm of heterogeneous agents, it is shown in [20] that flocking and segregation can be realized by adjusting magnitudes of interaction potentials between different types of agents. Besides, the seed growing graph partition (SGGP) algorithm is given as a splitting/merging strategy in [15] to solve the multiple targets tracking problem in mobile sensor network.

In this paper, we consider the problem of dynamically splitting/rejoining control of an MAS consisting of a large number of identical agents with limited sensing range. We will first present a simple splitting/rejoining control law based on attraction-repulsion rules for interaction among agents. To be specific, The control law consists of long-range attraction and short-range repulsion interactions between a pair of coupled agents to guarantee the coherence and collision avoidance in the swarm, and a Gaussian-like repulsive interaction to split the swarm into clusters by varying certain parameter. We then apply the control law to multiple targets seeking problem. Specifically, a double targets seeking mission is completed with the control law in an MAS with 100 agents.

The rest of the paper is organized as follows. Section II gives the model description of the MAS. A swarm split control law is presented in Section III and further applied to multi-target seeking problems in Section IV. Section V
offers concluding remarks of the paper.

II. MODEL

![Diagram of two agents and their respective Frenet-Serret frames in the plane.]

We consider a system consisting of $N$ identical agents, each moving at unit speed and restricted to smooth motion in the plane. The agent motion is described by planar Frenet-Serret equations, which contains the evolution of the agent position and orientation for united-speed motion, as taken in [17], [21]. For example, the trajectories of agent $i$ and $j$ and their respective planar Frenet-Serret frames are shown in Figure 2. Consider $r_i(t)$ is the differentiable motion trajectory of the $i$th ($i = 1, 2, \ldots, N$) agent in the plane. Taking a positively oriented orthonormal frame $x_i$ and $y_i$ associated to $r_i(t)$, where $x_i$ is equal to the unit tangent vector $dr_i(t)/dt$, and $y_i = x_i^\perp$ is the unit normal vector positively oriented relative to $x_i$, we get the equation of the $i$th agents as

$$
\begin{align*}
\dot{x}_i &= x_i, \\
\dot{y}_i &= y_i u_i, \\
\dot{r}_i &= -x_i u_i.
\end{align*}
$$

(1)

For details of derivation see [17], [21]. The agents interact via a scalar curvature control law $u_i$. As the control law $u_i$ introduced in [21], let

$$u_i = \sum_{j \neq i} u_{ji},$$

(2)

with

$$
\begin{align*}
u_{ji} &= \left[ -\eta(|r_{ji}|) \left( \frac{r_{ji}}{|r_{ji}|} \cdot y_i \right) \left( \frac{r_{ji}}{|r_{ji}|} \cdot x_i \right) \\
&\quad + f(|r_{ji}|) \left( \frac{r_{ji}}{|r_{ji}|} \cdot y_i \right) + \mu(|r_{ji}|) x_j \cdot y_i \right],
\end{align*}
$$

(3)

where $r_{ji} = r_i - r_j$.

Usually, an agent just has a limited communication capacity. For implementation of the local coupling model, we take

$$\eta(|r_{ji}|) = \mu(|r_{ji}|) = \begin{cases} a & \text{if } |r_{ji}| < w, \\
0 & \text{otherwise}, \end{cases}
$$

(4)

where $w > 0$ is a given scaler specifying effective sensing range, and

$$f(|r_{ji}|) = C_1 \exp \left( -\frac{|r_{ji}|}{l_1} \right) - C_2 \exp \left( -\frac{|r_{ji}|}{l_2} \right),$$

(5)

The form of $f(|r_{ji}|)$ is a generalized Morse potential as discussed in [22]. The first and second terms in (5) are repulsive and attractive interaction between agent, respectively. Taking $C_1 > C_2 > 0$ and $0 < l_1 < l_2$, $f(|r|)$ is short-range repulsive and long-range attractive and decays at infinite distances, as would be expected for the vehicles with a limited communication range.

In the control law (3), the first term involving $\eta(|\cdot|)$ servers to orient each agent perpendicular to the baseline between the two agent, the second term involving $f(|r_{ji}|)$ servers to control inter-agent spacing, and the third term servers to align the heading directions of the agents. Under initial conditions that each agent is in interaction range of at least one other agent, the control law (3) does avoid collisions and maintain system cohesiveness, as discussed in [17], [21]. The final heading direction of the system is determined by group average motion and depends on initial condition.

![Graph of $f(\cdot)$ and $g(\cdot)$ in control law (3) and control law (6).]

III. SPLIT CONTROL LAW

The studies on multi-particle systems in physics show that the competition between the inter-particle repulsive and attractive forces may cause multiple clusters formations [23]. In this paper we use this principle to control swarm splitting. Besides long-range attractive and short-range repulsive interactions as commonly considered, a Gaussian-like repulsion term is introduced to split the cohesive formation of agents. We note that in studies of soft condensed matters, Gaussian-shaped pairwise interaction potential between particles is considered and brings about some quite interesting new phenomena [24].

The swarm split control law we adopt is described as follows

$$u_{ji}^S = u_{ji} + g(|r_{ji} \cdot y_j|) \left( \frac{r_{ji}}{|r_{ji}|} \cdot y_i \right),$$

(6)

with

$$g(|r_{ji} \cdot y_j|) = \lambda \exp \left( -\frac{(|r_{ji} \cdot y_j| - b)^2}{2\beta^2} \right).$$

(7)
where $\lambda \geq 0$ is the maximum value of $g(\cdot)$ and $b > 0$ is the location of the maximum of $g(\cdot)$, $\beta > 0$ controls the width of the “bell”. In this paper, we let the width of the Gaussian repulsion be about the same range of the attractive interaction as in (5).

Observe that for $h(|r_{ji}|) = f(|r_{ji}|) + g(|r_{ji}|)$, there are three typical ingredients: short-range repulsion, mid-range repulsion, and long-range attraction, as shown in Figure 3. The short-range repulsion ensures the avoidance of overlap between agents and the long-range attraction makes the agents converge. The mid-range repulsion caused by $g(\cdot)$ enables agents to tend to move along the direction in which the resultant repulsive force is decreasing. For the “bell” shape of the Gaussian function as shown in Fig. 3, there are two possible directions in which the repulsive force is decreasing. One is with the increase of the relative distance between agents when $r < b$, and the other with the increase of the relative distance when $r > b$. The motion along the first direction leads to aggregation of the agents locating within a small vicinity, whereas the motion along the second direction leads to separation of agents in a comparatively large scale. These two effects render a coherent swarm to split into small clusters or subswarms eventually. In addition, the term $|r_{ji} \cdot \mathbf{y}_j|$ in (7) serves to restrict the swarm splitting in the direction perpendicular to the common heading direction of the swarm.

Assume that there are $M$ clustered subswarms in the final formation of agents. Let $\mathcal{N}_k$ be the index set of the agents belonging to the $k$th clustered subswarm and $n_k$ be the number of agents of the $k$th subswarm for $k = 1, 2, \ldots, M$. The center of the $k$th subswarm is defined as follows

$$s_k = \frac{1}{n_k} \sum_{j \in \mathcal{N}_k} r_j. \quad (8)$$

Then the relative distance between the $k, j$th subswarms is considered as $s_{lk} = s_k - s_j$, and the radius of the $k$th subswarm $\rho_k$ that is defined as the maximum of the distances between the center and the members of the subswarm.

Generally speaking, the larger the value of $\lambda$ in (7) is, the more clusters appear in the swarm, because of the large magnitude of the Gaussian-like repulsion as shown in Figure 3. We call $\lambda$ the splitting parameter, and take $0 \leq \lambda < f(0)$. Particularly, for $\lambda = 0$, $u_{ji}^a = u_{ji}$, the split control law is then the same as the steering control law (3). The parameter $b$, referred to as the expanding parameter, is the location of the maximum value of the Gaussian-like repulsion (7). It can affect the relative distances $s_{lk}$ between separated subswarms. Our simulation results show that $s_{lk} > b$, i.e., the distance between subswarms is always greater than $b$.

It can be understood that the repulsion between subswarms reaches maximum when their relative distance is about $b$ (the distance at which the Gaussian-like repulsion between agents reaches maximum). In final formation of the system, if $s_{lk} < w$, i.e., the relative distances between subswarms are smaller than the communication range of agents, the separated subswarms can be aligned to a common heading direction by the alignment interaction in the steering control law (3). Otherwise, for $s_{lk} > w + \rho_k + \rho_l$, i.e., no one agent of the $k$th subswarm locates in the communication range of any agents in the $l$th subswarms, the separated subswarms may move along different directions independently.

IV. SIMULATION RESULTS

We present simulation results of the swarm model with split control law (6). The system consists of $N = 100$ agents. Each agent represents a vehicle moving in the plane at unit speed. The following parameters remain fixed throughout all simulations: $a = 0.05$, $w = 3$ for the functions $\eta(\cdot)$ and $\mu(\cdot)$, $C_1 = 2, l_1 = 0.5, C_2 = 1, l_2 = 1$ for the attraction/repulsion interaction $f(\cdot)$, $b = 1.39, \beta = \sqrt{5}/10$ for the Gaussian-like repulsion $g(\cdot)$. All simulations begin with $\lambda = 0$, so the initial control is simply implemented by the steering control law (3). The initial positions of the 100 agents are uniformly chosen at random from the box $[-1, 1]^2$, and the initial orientation $\theta_i$ of each agent is uniformly chosen at random from $\theta_i \in [-\pi/4, \pi/4]$.

A. Splitting/rejoining maneuver with changing of $\lambda$

Typical clusters formations and split/rejoin maneuvers of the swarm system ($N = 100$) at different values of the splitting parameter $\lambda$ are demonstrated in follows with on-the-fly parameter modification.

Figure 4 shows the system’s split/rejoin maneuver for the splitting parameter $\lambda = 0.29$. Near $t = 20$, the splitting parameter is switched off, a single swarm reforms.

Figure 4 shows the system’s split/rejoin maneuver for the splitting parameter $\lambda = 0.29$. Near $t = 20$, $\lambda$ increases to
0.29, and the split control law (6) is switched on. After then the cohesive swarm begins to split into two subswarms, each of which consists of 50 agents. It can be understood that with the effect of the Gaussian-like repulsion (7), every pair of agents located symmetrically with the common heading direction are pushed to fly to depart from each other. At $t = 35$, the distance between the two subswarms is $s_{12} = 1.77$, and clearly, $s_{12} > b$. Then the two subswarms keep the relative distance until the splitting parameter $\lambda$ is switched off at about $t = 40$. As $\lambda$ is decreased to zero, the control law reverts to (3), and the two subswarms reunite into one again. Figure 5 shows that the symmetry of subswarms in number of agents holds for different initial conditions. The differences between the agent numbers of two subswarms $|n_1 - n_2|$ is very small compared to the total agent number $N = 100$ in 20 simulations for random initial conditions. This symmetry depends on the symmetry of pairwise interactions between agents and the symmetry of the formation at the beginning of the splitting process.

![Fig. 5. Number differences between two subswarms separated with the split control law (6) for $\lambda = 0.29$ in 20 simulations (N=100).](image)

For the splitting parameter $\lambda = 0.49$, Figure 6 shows that one cohesive swarm can split into six separate subswarms with the split control law (6) switched on at near $t = 20$. The clusters formation is also approximately symmetric along the common heading direction of agents. After $\lambda$ is decreased to zero at near $t = 40$, the six subswarms reform as one swarm.

The one-dimensional phase diagram for the splitting parameter $\lambda$ of the swarm system with $N = 100$ is shown in Figure 7. In region $C_1$ where $0 \leq \lambda \leq 0.25$, one cohesive swarm forms. The regions $C_2 : \{0.25 < \lambda \leq 0.34\}$, $C_3 : \{0.34 < \lambda \leq 0.42\}$, $C_4 : \{0.42 < \lambda \leq 0.45\}$ and $C_6 : \{0.45 < \lambda \leq 0.49\}$ correspond to the 2, 3, 4, 6 clusters formations respectively. If $\lambda$ increases to $\lambda = 0.5$, some agents are observed to escape from a local neighborhood of the swarm, and thus no longer interacts with other agents. The boundaries of the regions are not critical, because the phase transitions are gradual. Generally, the formations are symmetric along the moving direction of the swarm, and the clustered subswarms move parallel to each other and keep approximative equal relative distances between neighboring subswarms for $0.25 < \lambda \leq 0.49$.

**B. Formation expansion by changing of $b$**

The following simulations show the effect of the expanding parameter $b$ in (7). After the clusters formed, changing the expanding parameter $b$ makes differentiation in the relative distances between subswarms in the steady state. As before, the splitting parameter is initially set as $\lambda = 0$, and it is switched on at about $t = 20$.

Figure 8 shows expanding of the relative distance between two subswarms with the increase of expanding parameter $b$. To begin with, the cohesive swarm splits into two subswarms as $\lambda$ is increased to 0.29 at near $t = 20$. After then, at about $t = 40$, $b$ is increased to $b = 2.39$, and the two subswarms begin to fly to depart from each other, because the maximum location of the Gaussian-like repulsion shifts. The response curve of the separation between two subswarms shown in Figure 8 reveals that the relative distance is always greater than $b$. After the extending process, the two subswarms are aligned to a same heading direction again and keep their relative distance $s_{12} = 3.14$ in motion. Figure 9 shows that, if $b$ increases to $b = 3.39$ at about $t = 40$, the separated subswarms become independent. They move along different directions since the subswarms are not in the communication range of one another. The transition threshold from the clusters keeping constant relative distance to the independent clusters moving along different directions is about $b = 2.45$. 

![Fig. 6. The splitting and rejoining of the swarm system with the split control law (6) for $\lambda = 0.49$. The upper figure is a snapshot of the clusters formation and the agents' trajectories in the $r = (x, y)$ plane, the center plot shows $y$ vs $t$, and the lower plot, $\lambda$ vs $t$. Agents start out in random initial positions and directions. The splitting parameter $\lambda$ is switched on $\lambda = 0.38$ at near $t = 20$, and is switched off at near $t = 40$.](image)

![Fig. 7. Phase diagram of the swarm system with $N = 100$ for the splitting parameter $\lambda$.](image)
V. MULTIPLE TARGETS SEEKING

The split control law given by (6) gives a starting point for the design of formation controller for complex tasks. The swarm system with local coupling can automatically split into a few subswarms by changing the split parameter \( \lambda \). The separated subswarms can be turned to independent through modification of the expanding parameter \( b \), so that they can be independently redirected to complete different missions, e.g., to seek different targets. We now demonstrate how to code a basic multiple target seeking behavior into the split control law to guide the clusters formation of the swarm system. The approach discussed in this section is similar in spirit to that presented in [17], [25], where the attraction forces between the agents and targets are introduced with global coupling.

A. Double targets seeking law

The goal of the control design is to split the swarm into two independent subswarms with similar agent numbers, and each subswarm can seek a different fixed target. The targets considered here are two fixed points in the plane. The information of the positions of the targets can be known by agents. Denote the locations of the two targets by \( \bar{r}_1 \) and \( \bar{r}_2 \).

The double targets seeking control law is given as

\[
\begin{align*}
&u^T_{ji} = \sum_{j \neq i} \left[ f(|\bar{r}_{ji}|) \left( \frac{\bar{r}_{ji}}{|\bar{r}_{ji}|} \cdot \gamma y_i \right) \\
&+ g(|\bar{r}_{ji} \cdot y_j|) \left( \frac{\bar{r}_{ji}}{|\bar{r}_{ji}|} \cdot \gamma y_i \right) \\
&+ \gamma \left[ 1 - \left( \frac{r_0}{|\bar{r}_{ik}|} \right) \left( \frac{\bar{r}_{ik}}{|\bar{r}_{ik}|} \cdot \gamma y_i \right) \right]
\end{align*}
\]  

(9)

where \( f(\cdot) \) and \( g(\cdot) \) are same as in (5) and (7) respectively, \( k \in \{1, 2\} \) is the index of the target nearest to the \( i \)th agent, \( \bar{r}_{ik} = \bar{r}_i - \bar{r}_k \) is the vector directed form the position of the \( i \)th agent to the position of its nearest target, \( \gamma \) is a weighting constant, \( r_0 \) is a positive parameter. The first term in (9) provides for coherence and collision avoidance. The second term is a Gaussian-like repulsion for swarm splitting. The third term, which is global and similar to that taken in [17], serves to steer agents toward their nearest target.

B. Simulation result

Figure 10 shows the result of a simulation of the swarm system with double targets seeking control law (9). The two targets are located at \( \bar{r}_1 = (50, 6) \) and \( \bar{r}_2 = (50, -6) \) respectively. There are \( N = 100 \) agents in the swarm system. The initial positions of the 100 agents are uniformly chosen at random from the box \([-1, 1]^2\), and the initial orientations \( \theta_i \) of each agent is uniformly chosen at random from \( \theta_i \in [-\pi/4, \pi/4] \). The following parameters in this simulations is: \( C_1 = 2, t_1 = 0.5, C_2 = 1, t_2 = 1 \) for the attraction/repulsion interaction \( f(\cdot) \), \( \beta = \sqrt{5}/10 \) for the Gaussian-like repulsion \( g(\cdot) \), \( \gamma = 0.1, r_0 = 0.1 \) for the control law (9). The splitting parameter \( \lambda \) and the expanding parameter \( b \) are initially \( \lambda = 0 \) and \( b = 1.39 \) respectively. Near \( t = 10 \), \( \lambda \) is...
increased to 0.29, and the cohesive swarm splits into two subswarms. Near \( t = 20 \), \( b \) is increased to 3.39 so that \( t = 20 \). Subsequently, the cohesive swarm splits into two subswarms. Near \( t = 20 \), \( b \) is increased to 3.39 so that \( t = 20 \). A swarm split control approach whereby a cohesive swarm can split into several subswarms is presented in this paper. The two red points are the targets. The lower plot shows \( \lambda, b \) and the separation between the two clustered subswarms vs \( t \).

VI. CONCLUDING REMARKS

A swarm split control approach whereby a cohesive swarm can split into several subswarms is presented in this paper. This approach is only based on local interactions between agents and is symmetric in that it does not distinguish agents of different types. It is robust in the sense that the formation transition of the system can be completed even though some agents are disabled. The final formations of clusters are approximately symmetric with respect to the moving direction of the swarm. The clustered subswarms can be designed to cooperate to complete complex tasks, for example, multiple target seeking problem in which the subswarms converge to given different target points. The split control law is not only valid for the Morse potential-like interaction between agents as considered in this paper, but is also valid for other types of interactions, such as the inverse power force \( f(r) = A/r - B/r^2 \). Moreover, it also can be applied to agents governed by classical Newton’s Law of Motion without the unit speed limitation, and leads to similar results.

REFERENCES


